$\bigcup \mathcal{A} = \bigcup_{A \in \mathcal{A}} A$

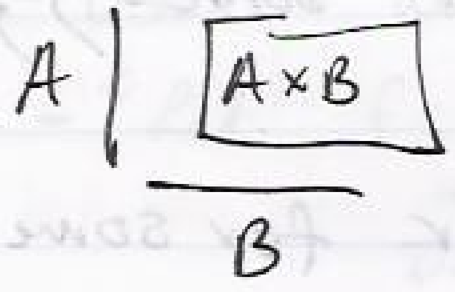
$= \{p \mid \text{for some } A \in \mathcal{A},\ p \in A\}$

$\Rightarrow \bigcup_{A \in \mathcal{A}} B_A = \{p \mid \text{for some } A \in \mathcal{A},\ p \in B_A\}$

Ex: $\bigcup_{A \in P(B)} \{A, B\}$ ← random example

Practice: write out $\bigcup_{A \in P(\{0,1\})} \{A, 0\}$

- $A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$



Aug 26, 2010 Thursday

Hwk #1 due Sept. 6, 6PM, CH 313

§1 #3, 5, 10

§2 # 2, 4, 5

§3 #5, 13

§4 #5, 8, 11

§1 Sets

§2 functions

§3 relations

§4 reals and integers

Functions

A rule of assignment is a subset r of $A \times B$ (where A, B are sets) such that

$$[(a,b),\ (a,b') \in r] \Rightarrow b = b'$$

for all $a \in A$ and $b, b' \in B$.

These are also called partial functions from A to B.

[redacted] $\{(x, 1/x) \mid x \in \mathbb{R} - \{0\}\}$

is a rule of assignment.

If r is a rule of assignment, define

domain $r = \{a \mid (a,b) \in r \text{ for some } b\}$,

and $\underbrace{\text{image } r}_{\text{"image set"}} = \{b \mid (a,b) \in r \text{ for some } a\}$

other authors may use "range"

Let $q = \{(x, 1/x) \mid x \in \mathbb{R} \setminus \{0\}\}$

$\Rightarrow$

Claim: image $g = \mathbb{R} - \{0\}$

Proof $\Rightarrow$ If $p \in$ image of g, then $p = 1/x$ for some $x \in \mathbb{R} - \{0\}$, so $p \in \mathbb{R} - \{0\}$ because non-zero reals have nonzero reciprocals. Therefore, the image $g \subseteq \mathbb{R} - \{0\}$. If $p \in \mathbb{R} - \{0\}$, then $(\frac{1}{p}, \frac{1}{1/p}) \in g$, so $\frac{1}{1/p} = p$ and $\frac{1}{1/p} \in$ image g, so $p \in$ image of g.

Therefore, $\mathbb{R} - \{0\} \subseteq$ image g. Thus, $\mathbb{R} - \{0\} =$ image g. □

Shorter version of proof $\Rightarrow$ Every reciprocal of a non zero real is a non zero real, and every nonzero real is a reciprocal of a nonzero real, namely its reciprocal. □

- A function is a rule of assignment ~~and~~ a together with $\boxed{\text{super set of its image set.}}$ "range" "codomain"

- g together with $\mathbb{R}$ is a function.
- Usual notation for functions:
  - "Let $f: A \to B$ where $f(x) = \dots$ for all $x \in A$."

    Domain (for A); book calls B the range, also called codomain

- Restrictions of functions:

If $f: A \to B$ and $A_0 \subset A$, then the restriction of f to A_0 is written as $f|A_0$ or $f\restriction_{A_0}$ and is $\{(a, f(a)) \mid a \in A_0\}$

$f: \mathbb{N} \to \mathbb{N}$, $f(x) = x^2$

$f|_{\{1,2\}} = \{(1,1),(2,4)\}$

Composition of functions:

If $f: A \to B$ and $g: B \to C$, then define $g \circ f : A - C$ is the function defined by $(g \circ f)(a) = g(f(a))$.

- $f: A \to B$ is <u>injective</u> (or 1-to 1) if $f(a_1) = f(a_2) \Rightarrow a_1 = a_2$

For all $a_1, a_2 \in A$.

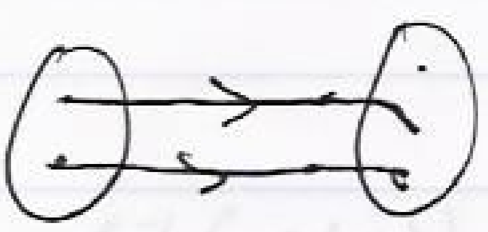
(Contrapositive: $a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$)

- $\ln : \underbrace{\mathbb{R}_+}_{\{x \in \mathbb{R} \mid x > 0\}} \to \mathbb{R}$ is surjective, meaning the codomain is also the image set

In detail, $f: A \to B$ is surjective if every $b \in B$ equals $f(a)$ for some $a \in A$.

<u>bijective</u> = injective and surjective

- not injective
- surjective



- injective
- not surjective

$\Rightarrow$

If $f: A \to B$ and $E \subset A$, then $f(E)$, or the image of E, is $\{f(x) \mid x \in E\}$, which

is the SAME as

$$\{y \in B \mid \underbrace{(x,y) \in f}_{f(x)=y} \text{ for some } x \in E\}.$$

- If $f: \mathbb{R} \to \mathbb{R}$ and $f(x) = x^2$, what is $f([-2,3))$? $\Rightarrow [0,9)$.

What is $f^{-1}([0,9])$?

Pre image: If $f: A \to B$ and $E \subset B$, then $f^{-1}(E)$, or the pre image of E, is:

$$\{x \in A \mid \underbrace{(x,y) \in f}_{f(x)=y}, \text{ for some } y \in E\}.$$

$\underbrace{(-3,3)}_{\text{open interval}}$ $\Rightarrow$ $\{(-3,3) =]-3,3[\}$ = Alternative notations

not an ordered pair

$\{$ordered pair: $(-3,3) = \langle -3,3 \rangle \}$ alternative notations

In general, $f^{-1}(f(E)) \neq E$ and $f(f^{-1}(G)) \neq G$

If $f: A \to B$, $E \subset A$, $G \subset B$, then

$f^{-1}(f(E)) \supset E$ and $f(f^{-1}(G)) \subset G$