T 9/7/10

Homework 1A

§1 #2, #6

§2 #1, #3

§3 #3, #12

§4 #4, #6

[redacted] [redacted] [redacted]

Due 9/15/10 10:00am

HW 1

§1: 3, 5, 10

§2: 2, 4, 5

§3: 5, 13

§4: 5, 8, 11

Idea: use

$(a+b)+1 = a+(b+1)$

$= (a+1)+b$

etc...

§4

#5a Prove by induction that if $a, b \in \mathbb{Z}_+ \Rightarrow a+b \in \mathbb{Z}_+$

Recall: $\mathbb{Z}_+ = \bigcap \{A \subset \mathbb{R} \mid 1 \in A \text{ and } (n \in A \Rightarrow n+1 \in A)\}$

Fact: $1 \in \mathbb{Z}_+$ and $(n \in \mathbb{Z}_+ \Rightarrow n+1 \in \mathbb{Z}_+)$

Try proof by [redacted] induction:

Claim: for all $a \in \mathbb{Z}_+$, for all $b \in \mathbb{Z}_+$,
$a + b \in \mathbb{Z}_+$ [redacted]

Base case: Subclaim: for all $b \in \mathbb{Z}_+$,
$1 + b \in \mathbb{Z}_+$.

Proof: $b \in \mathbb{Z}_+ \overset{A}{\Rightarrow} b + 1 \in \mathbb{Z}_+$.
So, $1 + b \overset{B}{=} b + 1 \in \mathbb{Z}_+$. $\square$

Induction step: Subclaim: Assuming $a \in \mathbb{Z}_+$ and $a + b \in \mathbb{Z}_+$ for all $b \in \mathbb{Z}_+$, also $(a+1) + b \in \mathbb{Z}_+$ for all $b \in \mathbb{Z}_+$.

Proof: $b \in \mathbb{Z}_+ \Rightarrow a + b \in \mathbb{Z}_+ \overset{A}{\Rightarrow} (a+b) + 1 \in \mathbb{Z}_+$.
So, $(a+1) + b \overset{BC}{=} (a+b) + 1 \in \mathbb{Z}_+$ $\square$

This proves the claim by induction on a $\square$.

(A) $\mathbb{Z}_+$ is inductive.
(B) $\mathbb{R}$, + is commutative
(C) $\mathbb{R}$, + is associative

T 9/7/10

§3 #5a) $S = \{(x,y) \mid y = x+1 \text{ and } 0 < x < 2\}$
$S' = \{(x,y) \mid y - x \in \mathbb{Z}\}$
★ both $(x,y) \in \mathbb{R} \times \mathbb{R}$

- Show S' is an equivalence relation
- Show $S' > S$
- Describe the equivalence classes of S'.

Reflexivity: $x \in \mathbb{R} \Rightarrow x - x = 0 \in \mathbb{Z} \Rightarrow (x,x) \in S'$

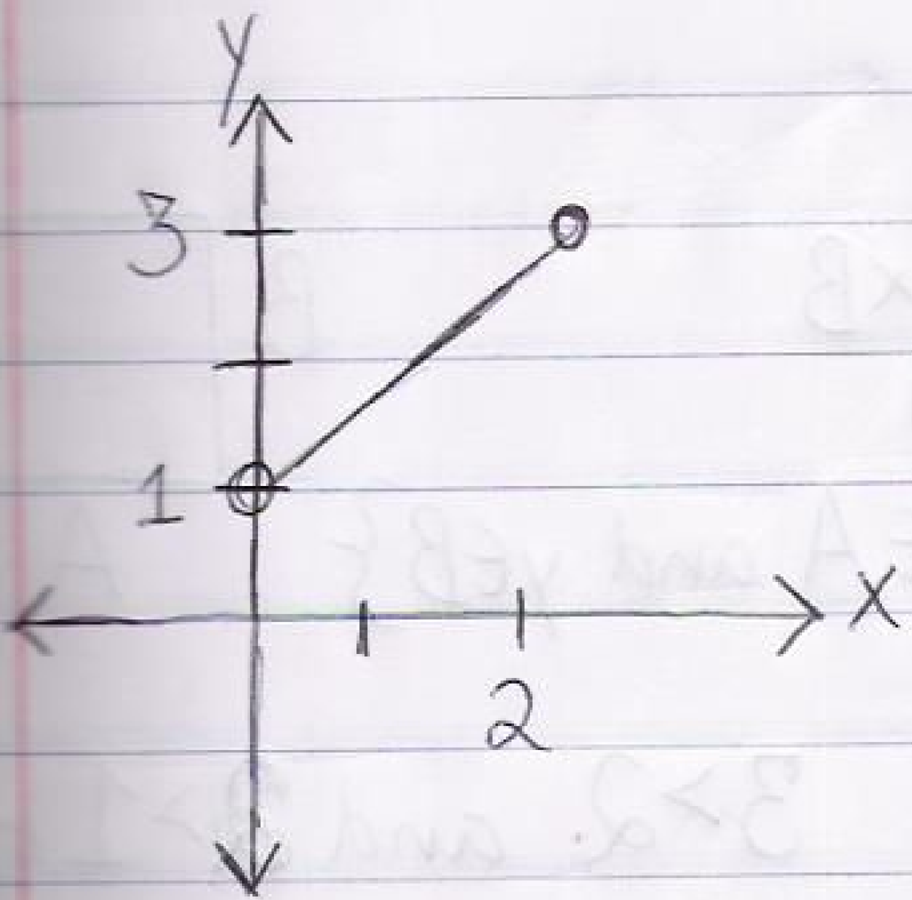
Transitivity: $(x,y), (y,z) \in S' \Rightarrow$ there exist $m, n \in \mathbb{Z}$ such that $y - x = m$ and $z - y = n \Rightarrow z - x = (z-y) + (y-x) = n + m \in \mathbb{Z} \Rightarrow (x,z) \in S'$.

Symmetry: $(x,y) \in S' \Rightarrow$ there exists $m \in \mathbb{Z}$ such that $y - x = m \Rightarrow x - y = -(y-x) = -m \in \mathbb{Z} \Rightarrow (y,x) \in S'$.

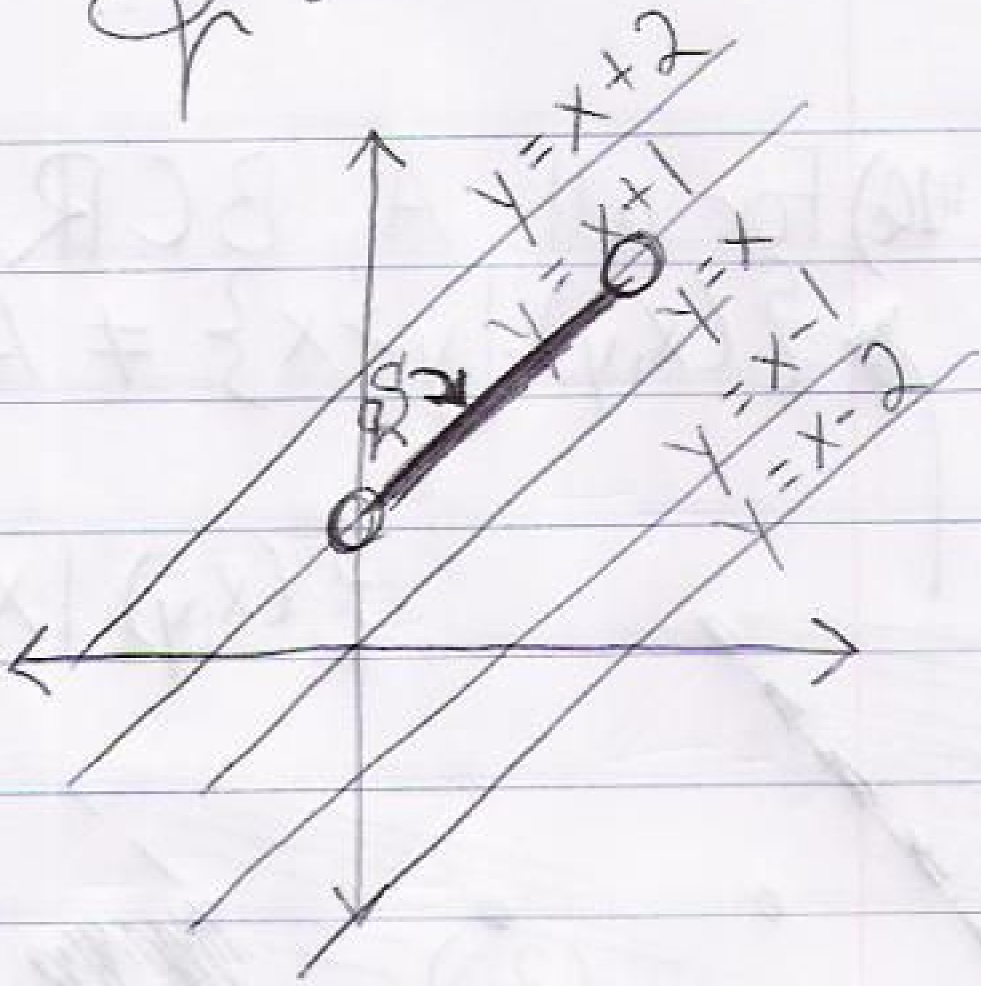
! $S' = \bigcup_{m \in \mathbb{Z}} \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid \underbrace{y - \cancel{x} = m}_{\substack{y = m + x \\ \text{or} \\ y = x + m}}\}$

T 9/7/10

S_r:



S'_r:



$(x,y) \in S_r \Rightarrow y = x+1 \Rightarrow y - x = 1 \in \mathbb{Z} \Rightarrow (x,y) \in S'_r$

For any $x \in \mathbb{R}$, the S'_r-equivalence class determined by x is $\{y \mid x S'_r y\}$
or
$\{y \mid (x,y) \in S'_r\}$

The S'_r-class of 0 is $\{y \mid \underbrace{y - 0 \in \mathbb{Z}}_{0 S'_r y}\}$

$= \{y \mid y \in \mathbb{Z}\} = \mathbb{Z}$

The S'_r-class of $1/2$ is $\{\pm 1/2, \pm 3/2, \pm 5/2, \dots\}$

The S'_r-class of x is $\{\dots, x-2, x-1, x, x+1, x+2, \dots\}$

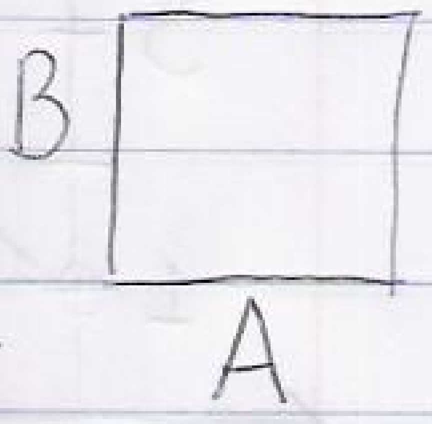
T 9/7/10

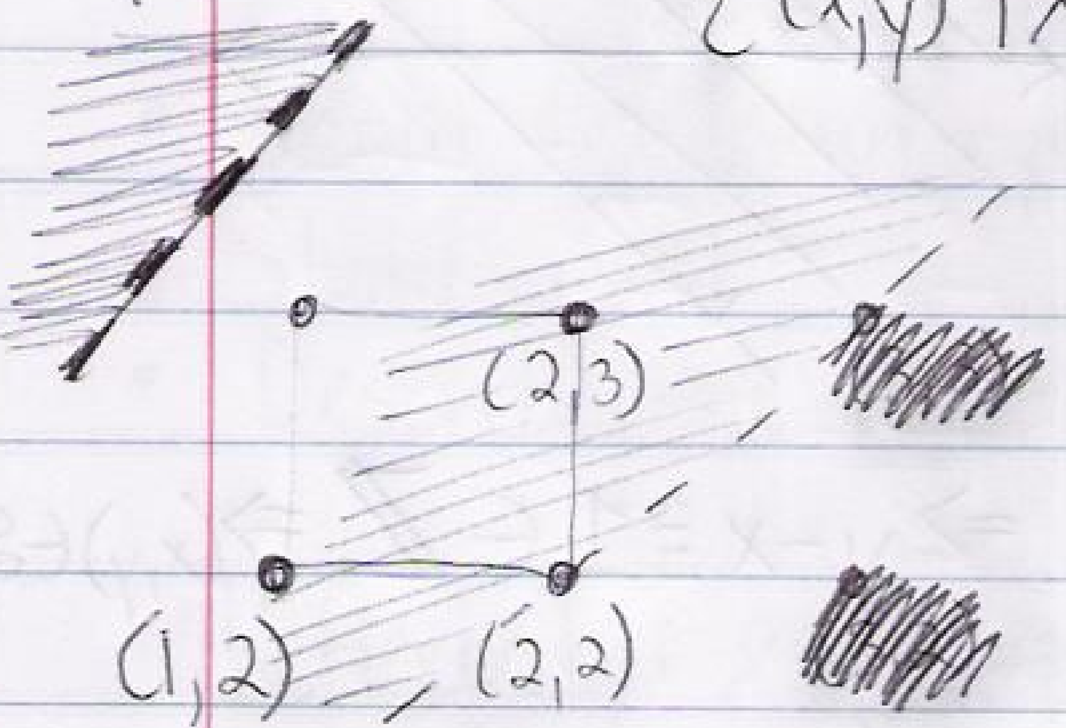
§1 #10c) For all $A, B \subset \mathbb{R}$

$$\{(x,y) \mid y > x\} \neq A \times B$$

$$\qquad\qquad\qquad\qquad \parallel$$

$$\{(x,y) \mid x \in A \text{ and } y \in B\}$$





$3 > 2$ and $2 > 1$

Suppose $A, B \subset \mathbb{R}$ and $\{(x,y) \mid y > x\} = A \times B$.

Then $(3,2) \in A \times B$.

$\Rightarrow 3 \in A$ and $2 \in B$

Also, $2 \in A$ and $1 \in B$

$\Rightarrow (2,2) \in A \times B$

But $2 \not> 2$.

$\rightarrow \leftarrow$

Contradiction!