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General Topology
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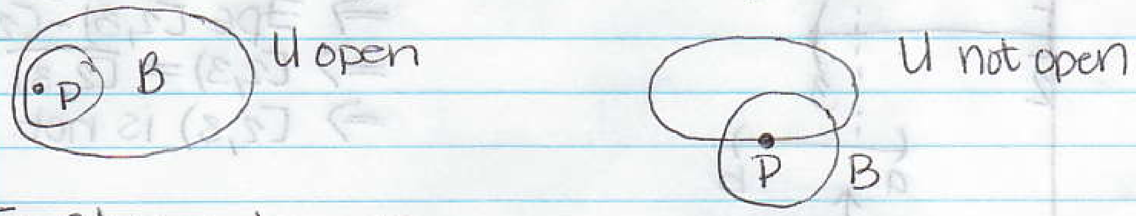
Review for Midterm 2:

$X_1 = \mathbb{R}$ has basis $\mathcal{B} = \{(a,b) \mid a, b \in \mathbb{R}\}$.

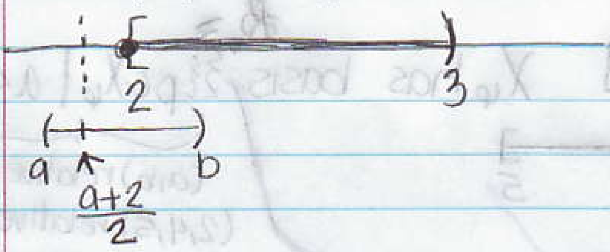
U is open $\iff \forall p \in U \ \exists B \in \mathcal{B} \ p \in B \subset U$.

U is not $\iff \exists p \in U \ \forall B \in \mathcal{B} \ (p \in B \Rightarrow B \not\subset U)$



★ $[2,3)$ is not open

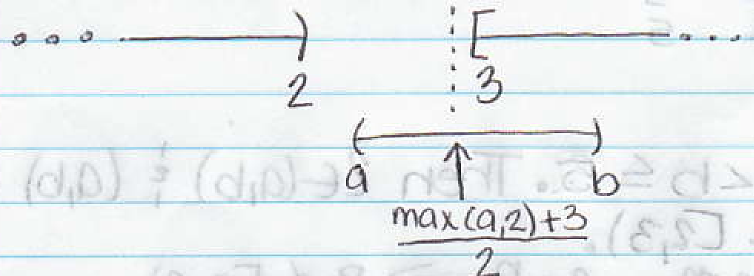
$2 \in (a,b) \Rightarrow a < 2 < b \Rightarrow (a,b) \not\subset [2,3)$



C closed $\iff X - C$ ~~closed~~ open

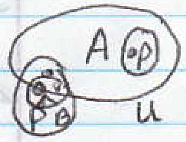
C not closed $\iff X - C$ not open

$[2,3)$ not closed $\iff \underbrace{(-\infty, 2) \cup [3, \infty)}_{\mathbb{R} - [2,3)}$ not open



$3 \in (a,b) \Rightarrow (a,b) \not\subset (-\infty, 2) \cup [3, \infty)$

For n big enough $2.\underbrace{99\ldots9}_{n} \in (a,b) - ((-\infty,2) \cup [3,\infty))$



$p \in \overline{A} \iff \forall U$ open $(p \in U \Rightarrow U \cap A \neq \emptyset)$

$p \in \overline{A} \iff [\forall U$ neighborhood of p, $U \cap A \neq \emptyset]$

$p \in \overline{A} \iff [\forall B \in \mathcal{B} \ p \in B \Rightarrow B \cap A \neq \emptyset]$

C closed $\iff C=\overline{C}$

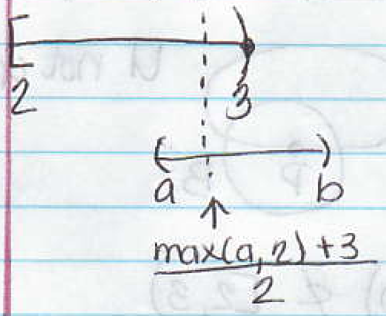
C not closed $\iff \exists p\in\overline{C}-C$

$3\in(a,b) \Rightarrow a<3<b \Rightarrow (a,b)\cap[2,3)\neq\emptyset] \Rightarrow 3\in\overline{[2,3)}$

$\Rightarrow \exists p\in\overline{[2,3)}-[2,3)$

$\Rightarrow [2,3)\neq\overline{[2,3)}$

$\Rightarrow [2,3)$ is not closed.



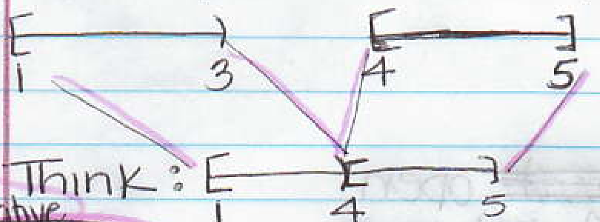
★ $X_6=[1,3)\cup[4,5]$ X_6 has basis $\mathcal{B}_0 = \{\{p\in X_6 \mid a<x<b\} : a,b\in$

$\underbrace{}_{(a,b)\text{ relative to } X_6}$

$(2,4.5)$ relative to X_6 =

$(2,3)\cup[4,4.5)$ relative to ℝ

Think:

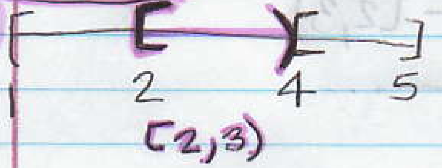


$(a,5]=(a,\infty)$ relative to X_6

$\cup\ \{\{p\in X_6 \mid a<x\} \mid a\in X_6\}$

$\cup\ \{\{p\in X_6 \mid x<b\} \mid b\in X_6\}$

$[1,b)=(-\infty,b)$ relative to X_6



Let $1\le a<2<b\le 3$. Then $2\in(a,b)$ & (a,b) is open, but $(a,b)\not\subset[2,3)$.

∴ Claim: $\forall B\in\mathcal{B}_0$ $2\in B \Rightarrow B\not\subset[2,3)$.

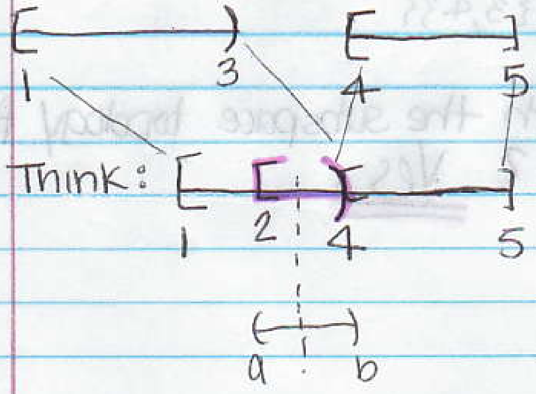
The same is true of (a,b) if $b\in[4,5]$

The same is true of $(a,5]$ & $[1,b)$

So, $[2,3)$ is not open in X_6.

$4\in\overline{[2,3)}-[2,3) \Rightarrow \overline{[2,3)}\neq[2,3) \Rightarrow [2,3)$ is not closed in X_6.

Why is $4 \in \overline{[2,3)}$?



$a, b \in X_6$

$4 \in \underbrace{(a,b)}_{\text{relative to } X_6} \Rightarrow (a,b) \cap [2,3) \neq \emptyset$

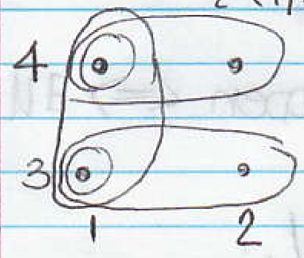
$(\underbrace{3.5}, 4.5)$ is not an open interval in X_6.

$3.5 \notin X_6$

★ $X = \{1, 2\}$ $\quad Y = \{3, 4\}$

$X \times Y = \{\underbrace{(p,q)}_{\text{ordered pair}} \mid p \in X \,;\, q \in Y\}$

$= \{(1,3), (1,4), (2,3), (2,4)\}$



$\mathcal{T}_X = $ 1 2

$\mathcal{T}_Y = $ 3 4

$\mathcal{T}_{X \times Y}$ has, by definition, basis $\mathcal{B} = \{U \times V \mid U \in \mathcal{T}_X \,;\, V \in \mathcal{T}_Y\}$

$= \{\emptyset \times \emptyset,\ \emptyset \times \{3\},\ \emptyset \times \{4\},\ \emptyset \times \{3,4\},\ \{1\} \times \emptyset,\ \{1\} \times \{3\},$
$\{1\} \times \{4\},\ \{1\} \times \{3,4\},\ \{1,2\} \times \emptyset,\ \{1,2\} \times \{3\},\ \{1,2\} \times \{4\},$
$\{1,2\} \times \{3,4\}$

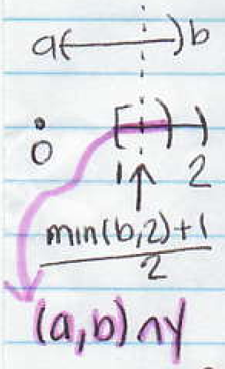
$= \{\emptyset, \overbrace{\{1\}\times\{3\}}^{\{(1,3)\}}, \{1\}\times\{4\}, \{1\}\times\{3,4\}, \{1,2\}\times\{3\},$
$\{1,2\}\times\{4\}, \{1,2\}\times\{3,4\}\}$

★ Let $Y = \{0\} \cup [1,2)$ with the subspace topology from $\mathbb{R}$.

- Is $\{0\}$ open in Y? **Yes.**

$\{0\} = \underbrace{\left(-\frac{1}{2}, \frac{1}{2}\right)}_{\text{open in } \mathbb{R}} \cap Y$

- Is $\{0, 1\}$ open in Y? **No.**



If $\{0,1\} = U \cap Y$, then, $1 \in (a,b) \subset U$ for some a, b in $\mathbb{R}$.

U open in $\mathbb{R}$

But then $(a,b) \cap (1,2) \neq \emptyset$, so $\{0,1\} \not\supset (a,b) \cap Y$.
$U \supset (a,b)$ so $\{0,1\} \not\supset U \cap Y$. This a contradiction.

- Is $\{0, 1\}$ closed in Y? **Yes**

When Y subspace of X, $V \subset Y$ is open $\iff \exists U$ open in X $V = U \cap Y$.

$Y - \{0, 1\} = (1,2) = \underbrace{(1,2)}_{\text{open in } \mathbb{R}} \cap Y.$

$\Rightarrow Y - \{0,1\}$ is open $\Rightarrow \{0,1\}$ is closed.