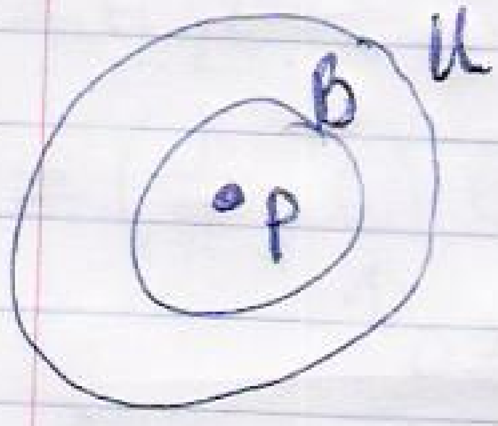
11/11/2010 – Notes Chapter # 25

Quiz question What is a locally connected space?

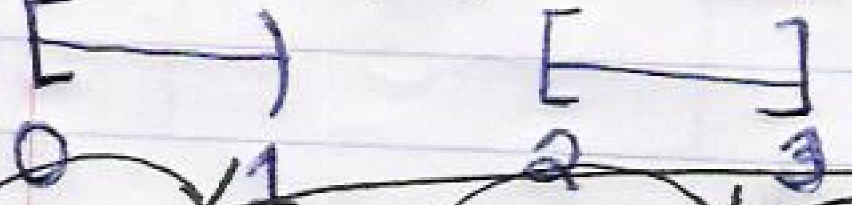
- A space X such that for every point $p \in X$, and every neighborhood U of p, p has a connected neighborhood $V \subset U$.

- Equivalently, a space with a basis $\mathcal{B}$ where every $B \in \mathcal{B}$ is connected.
  - Recall $\mathcal{B}$ is a basis for a space X if for every $p \in X$ and U neighborhood of p, $\exists B \in \mathcal{B}$ $p \in B \subset U$, and every $B \in \mathcal{B}$ is open.



- Equivalently, a space where every connected component is open.

$X = [0,1) \cup [2,3]$ subspace of $\mathbb{R}$.



the connected components of X.

$[0,1)$ and $[2,3]$ are intervals in $\mathbb{R}$, so they're connected.

x, y are in the same connected component if $x, y \in K$ for some connected $K \subset X$.

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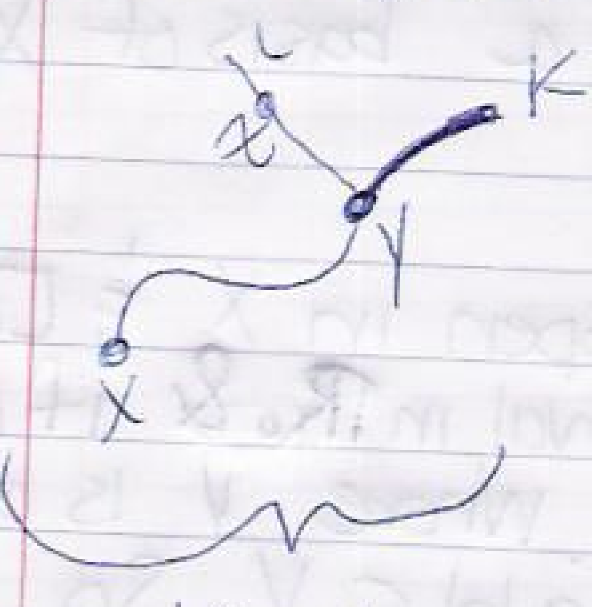
$\forall x, y \in [0,1)$ x, y are in the same connected component because $[0,1)$ is connected.

Likewise for all $x, y \in [2,3]$.
An equivalent definition of a connected component is, "a maximal connected subspace."

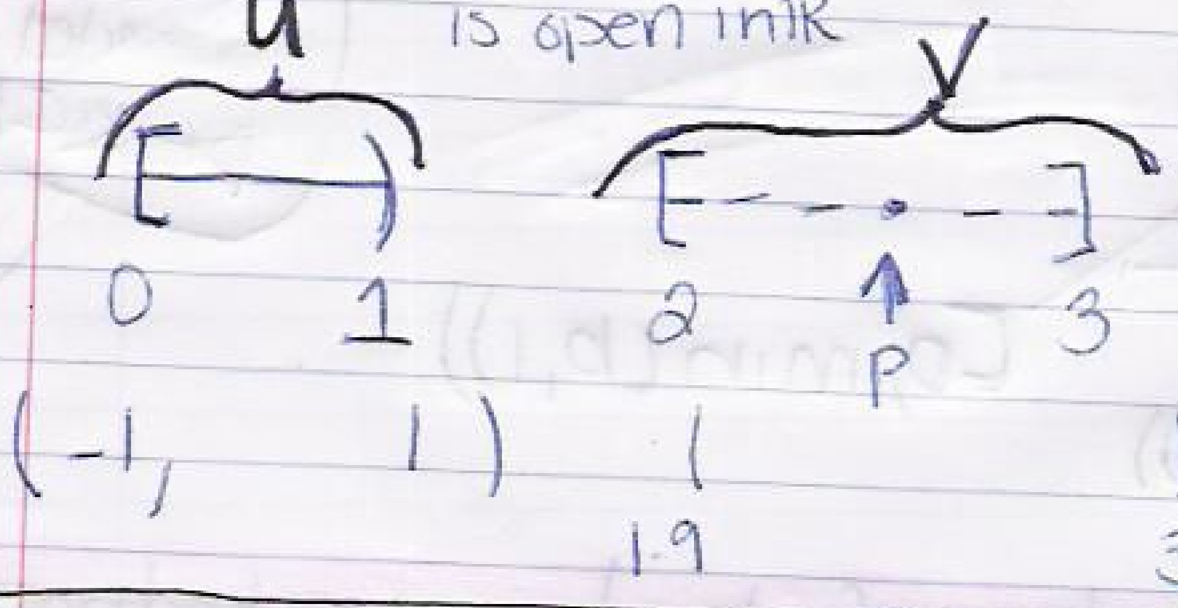


$K \cup L$

Suppose $[0,1) \subsetneq Y \subset X$. Then $\exists\, p \in Y - [0,1) = Y \cap [2,3]$

$\emptyset \neq U = [0,1) = \underbrace{(-1,1)}_{\text{open in } \mathbb{R}} \cap Y$ is open in Y.

$\emptyset \neq V = \underbrace{(1.9, 3.1)}_{\text{is open in } \mathbb{R}} \cap Y$ is open in Y



$U \cup V = Y$
By definition, Y is not connected.

$[0,1) = (-1,1) \cap X$ is open in X.
$[2,3] = \underbrace{(1.9, 3.1)}_{\text{open in } \mathbb{R}} \cap X$ is open in X.

All connected comp. of X are open, so X is locally connected.