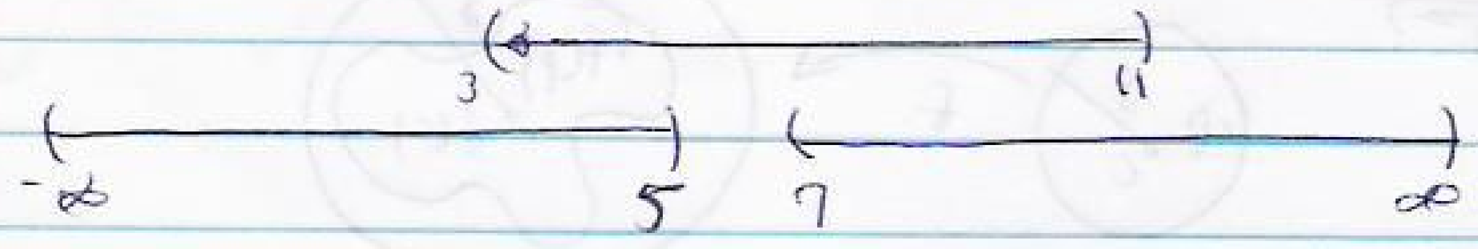
Jeffrey Castañeda

Notes for 11-18-10

What is the largest Lebesgue number of the open cover $\{(-\infty,5),(3,11),(7,\infty)\}$ of $\mathbb{R}$? 4



The answer for the warm-up solution is 0.16. Thus Lebesgue # is 0.16.

$\mathcal{U} = \{[0, 1-1/n) : n \in \mathbb{Z}_+\} \cup \{(0.84, 1]\}$.

Finite subcovers: $\mathcal{S}_n = \{[0, 1-1/n), (0.84, 1]\}$ for $n = 7, 8, 9, 10, \ldots$

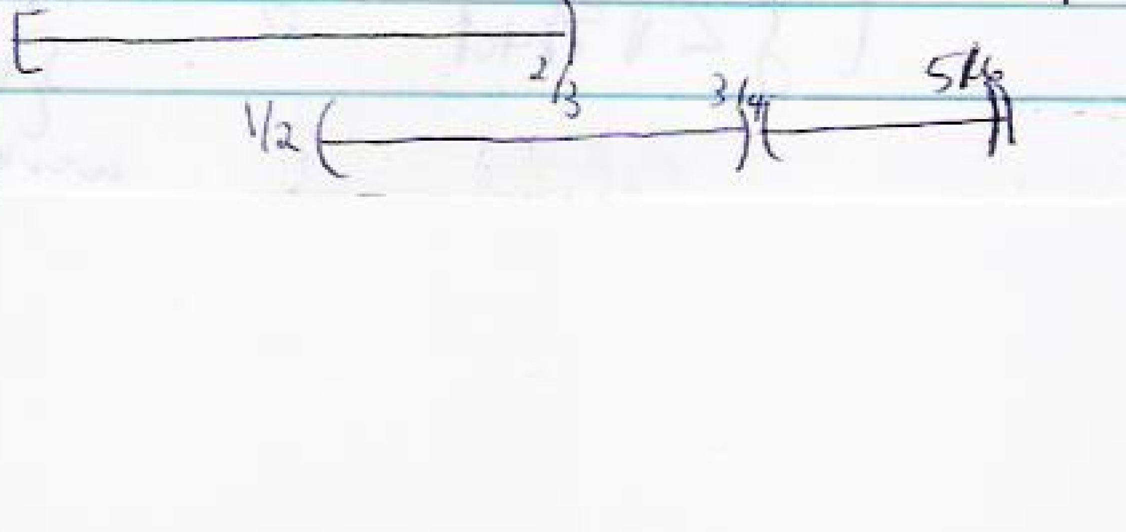
$\mathcal{S}_n$ has ~~subcover~~ Leb. # $1 - 1/n - 0.84$.

The Lebesgue # of $\mathcal{U}$ is the supremum of all the Lebesgue #'s of finite subcovers of $\mathcal{U}$.

$\lim_{n\to\infty}(1 - 1/n - 0.84) = 0.16$

$X = [0,1)$ $\quad \mathcal{U} = \{[0, 2/3), (1/2, 3/4), (2/3, 4/5), (3/4, 5/6), (4/5, 6/7), \ldots\} = \{[0, 2/3)\} \cup \{(\frac{n}{n+1}, \frac{n+2}{n+3}) : n \in \mathbb{Z}_+\}$

$\mathcal{U}$ has no finite subcover. $\mathcal{U}$ has no (positive) Lebesgue #



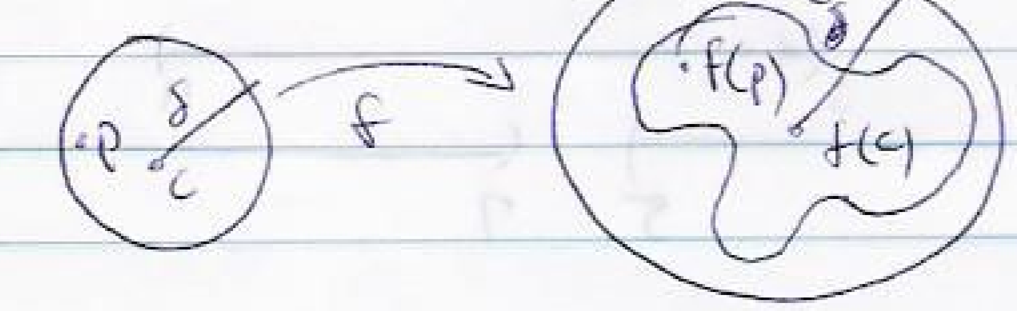
$\delta = 0.06$ fails: $\{4/5, 6/7\}$

$\Longleftrightarrow \forall U$ open $\subset Y$, $f^{-1}(U)$ open in X.

$f: (X, d_X) \to (Y, d_Y)$ is cts $\Longleftrightarrow \forall c \in X \ \forall \varepsilon > 0 \ \exists \delta_{c,\varepsilon} > 0$

such that $\forall p \in X \ [d_X(p,c) < \delta_{c,\varepsilon} \Rightarrow d_Y(f(p), f(c)) < \varepsilon]$

~~$f: (X, d_X) \to$~~



$f: (X, d_X) \to (Y, d_Y)$ is uniformly cts
$\Longleftrightarrow \forall \varepsilon > 0 \ \exists \delta_\varepsilon > 0 \ \forall c \in X \ \forall p \in X$
$[d_X(p,c) < \delta_\varepsilon \Rightarrow d_Y(f(p), f(c)) < \varepsilon]$

Example:

$f: \mathbb{R} \to \mathbb{R} \quad f(x) = 3x + 5$

uniformly continuous: $\delta = \varepsilon/3$ works

$g: \mathbb{R} \to \mathbb{R} \quad f(x) = x^2$

non-compact domain

continuous, but not uniformly continuous.

$d(p^2, c^2) = |p^2 - c^2| = |p-c||p+c| = |p-c||p-c+2c|$

want $< \varepsilon$ given $|p-c| < \delta$

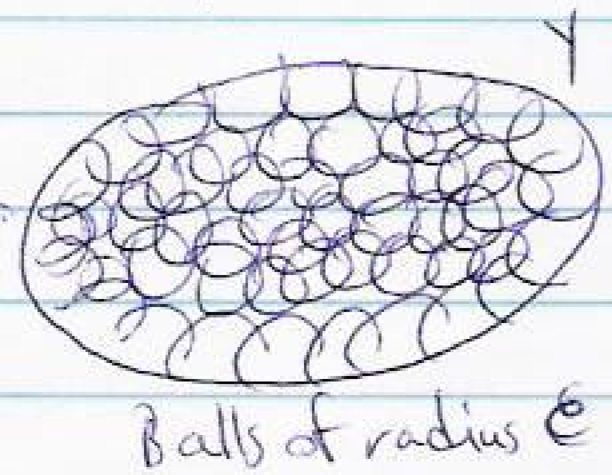
$|p-c||p-c+2c| \le |p-c|(|p-c| + |2c|) \le \delta(\delta + |2c|)$

$\le \delta \cdot 2\max\{\delta, |2c|\} = \max\{2\delta^2, |4c|\delta\}$.

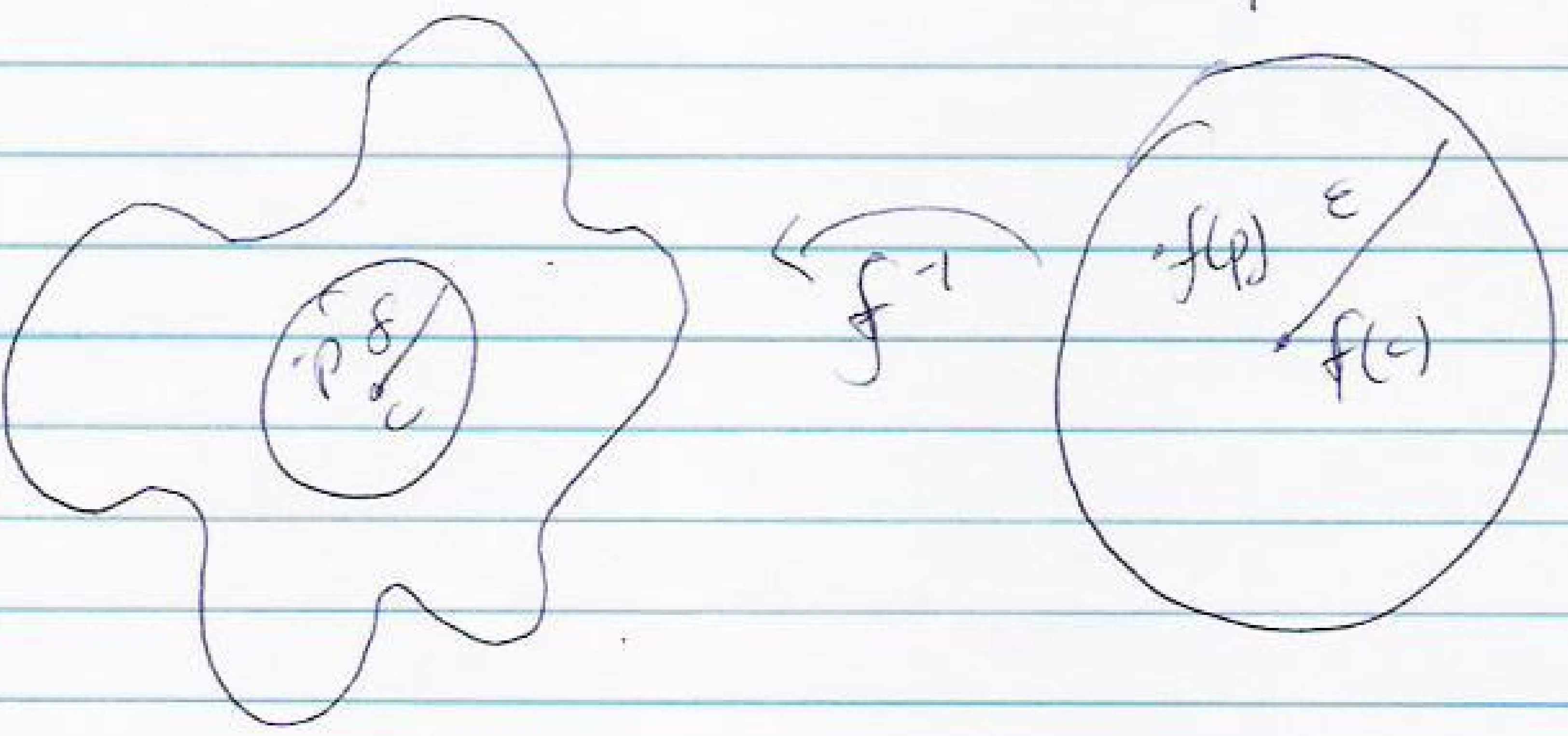
Need $\begin{cases} 2\delta^2 < \varepsilon \\ |4c|\delta < \varepsilon \end{cases} \Longleftrightarrow \begin{cases} \delta < \sqrt{\varepsilon/2} \\ \delta < \sqrt{\varepsilon/|4c|} \end{cases}$ $\quad \delta = 0.99 \cdot \min\left\{\sqrt{\varepsilon/2}, \sqrt{\varepsilon/|4c|}\right\}$ works.

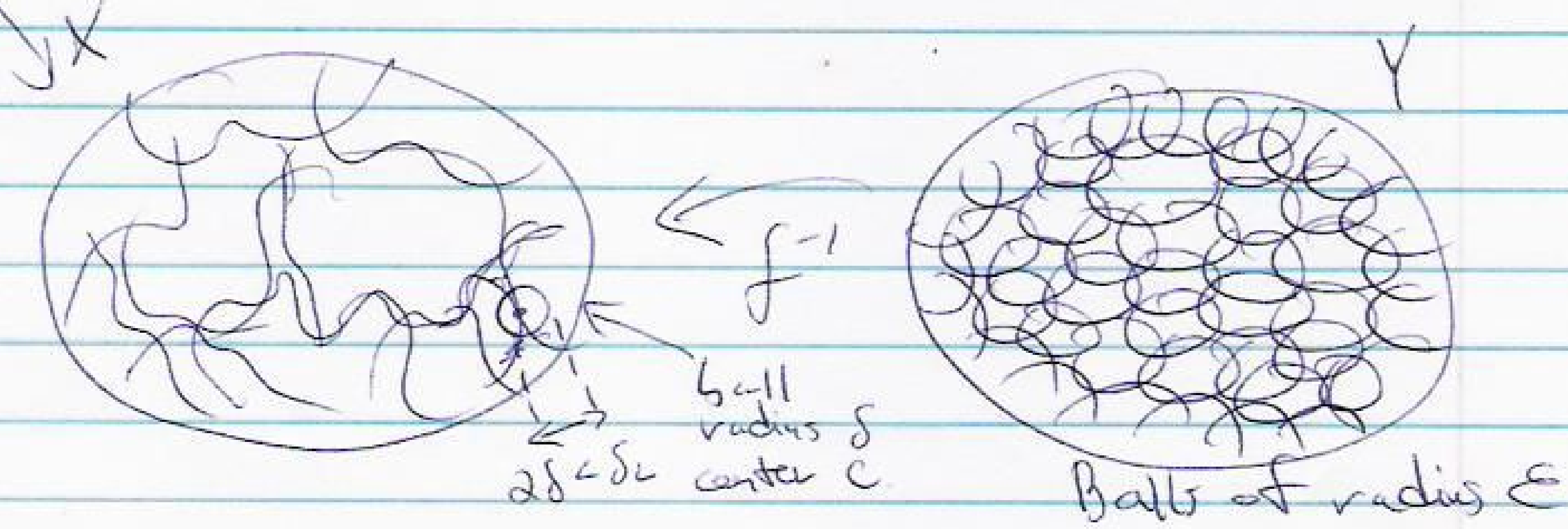
Notes for 11-18-10

If $f:(X, d_X) \to (Y, d_Y)$ is cts and X is compact, then f is uniformly cts.



Another, Picture of ~~Conti~~ Continuity





Open cover formed by set of preimages of ϵ-balls

δ_L = Lebesgue #

A subspace X of $\mathbb{R}^n$ is compact iff X is closed in $\mathbb{R}^n$

and has $\overbrace{\text{finite diameter}}^{\text{bounded}}$, ← for standard metric