HW3

(1) Define $\mathbb{Q}^{<\omega} = \bigcup_{n \in \mathbb{N}} (\mathbb{Q}^n)$.

Prove that there is a bijection from $\mathbb{Q}$ to $\mathbb{Q}^{<\omega}$.

Hint: If A & B have bijections from C, then there is a bijection from A to B.

(2) [Grad. only] Prove that if $(W, <)$ is well ordered and $x < y$ (in W), then x has an immediate successor in W. (See §3 for definition of immed. succ.)

Hint: consider the set $\{z \mid x < z\}$.

(3) Prove that if A is an uncountable set of positive reals, then $A \cap [\frac{1}{n}, \infty)$ is uncountable for some $n \in \mathbb{N}$.

Extra credit set theory project (worth 1 HW):

- Say that a set has cardinality $\aleph_1$ if it is uncountable but has a linear order such that every section is countable. (This is a definition.)
- Say that a set has cardinality $\aleph_{n+1}$ if it is uncountable and not of cardinality $\aleph_1, \aleph_2, \ldots,$ or $\aleph_n$, but does have a linear order such that every section is countable or of cardinality $\aleph_1, \aleph_2, \ldots,$ or $\aleph_n$.
- Prove that $\mathcal{P}(\mathcal{P}(\mathbb{R}))$ is not of cardinality $\aleph_1$ or $\aleph_2$ and that $\mathcal{P}(\mathcal{P}(\mathbb{R}))$ has a subset of cardinality $\aleph_3$.