

① Let  $\text{id}_X(a) = a$  for all  $a \in X$ .

HW 7

Let  $X_1 = X$  with topology  $\mathcal{T}_1$   
and  $X_2 = X$  with topology  $\mathcal{T}_2$ .

Prove that  $f = \text{id}_X : X_1 \rightarrow X_2$  is continuous  
if and only if  $\mathcal{T}_1$  is finer than  $\mathcal{T}_2$ .

② Let  $X = \{1, 2\}$  with base  $\{\{1\}, \{1, 2\}\}$   
and  $Y = \{4, 5, 6\}$  with base  $\{\{4, 5\}, \{5\}, \{5, 6\}\}$ .

How many of the nine functions from  $X$  to  $Y$   
are continuous? How many of the eight from  $Y$  to  $X$ ?

③ (Easy) Prove that if  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$   
are continuous, then  $g \circ f: X \rightarrow Z$  is continuous.

④ Give an example of  $f, g: \mathbb{R} \rightarrow \mathbb{R}$  such that  
 $f \circ g$  and  $g \circ f$  are continuous but  $f$  &  $g$  are not.

⑤ Prove that if  $a_1, a_2, a_3, a_4, \dots$   
converges to  $b$  in  $X$  and  $f: X \rightarrow Y$   
is continuous, then  $f(a_1), f(a_2), f(a_3), \dots$   
converges to  $f(b)$  in  $Y$ .

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