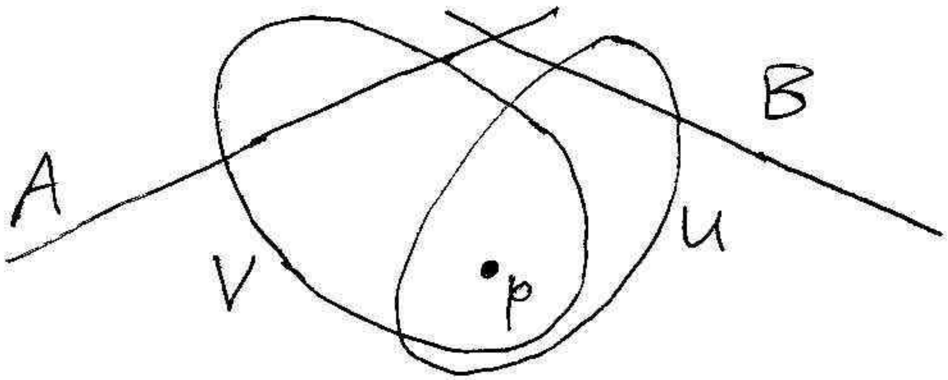
Proof of $\overline{A \cup B} = \bar{A} \cup \bar{B}$:

Proof of $\supset$: $\overline{A \cup B}$ is closed and $\overline{A \cup B} \supset A \cup B \supset A$. Since $\bar{A} = \bigcap \{ C \mid A \subset C \text{ \& } C \text{ closed} \}$, $\overline{A \cup B} \supset \bar{A}$. Likewise, $\overline{A \cup B} \supset \bar{B}$. Therefore, $\overline{A \cup B} \supset \bar{A} \cup \bar{B}$.

Proof of $\subset$: Suppose $p \in \overline{A \cup B}$. We will prove that $p \in \bar{A} \cup \bar{B}$. Seeking a contradiction, suppose $p \notin \bar{A} \cup \bar{B}$. Then $p \notin \bar{A}$. Hence, p has a neighborhood U disjoint from A. Likewise, p has a nbhd. V disjoint from B. Therefore, $U \cap V$ is a nbhd.* of p that is disjoint from $A \cup B$. Hence, $p \notin \overline{A \cup B}$. Contradiction!



* The intersection of two open sets is an open set.